1. In quale, tra gli intervalli indicati, è convessa la seguente funzione

$$f(x) = xe^{-6x^2+5}?$$

[a] $[-\frac{1}{2}, \frac{1}{2}]$ [b] $[-1, -\frac{1}{2}]$ [c] $[0, +\infty]$ [d] $[-\frac{1}{2}, 0]$ [e] $\mathbb{R}$

$$f(x) = x\,e^{-6x^2+5}$$

$$f'(x) = x' \cdot e^{-6x^2+5} + x \cdot \left(e^{-6x^2+5}\right)'$$

$$= e^{-6x^2+5} + x \cdot \left[e^{-6x^2+5} \cdot (-12x)\right]$$

$$= e^{-6x^2+5} - 12x^2 \cdot e^{-6x^2+5} = e^{-6x^2+5}\left(1 - 12x^2\right)$$

$$f''(x) = \left(e^{-6x^2+5}\right)' \cdot (1 - 12x^2) + e^{-6x^2+5} \cdot (1 - 12x^2)'$$

$$= \left[e^{-6x^2+5} \cdot (-12x)\right] \cdot (1 - 12x^2) + e^{-6x^2+5} \cdot (-24x)$$

$$= e^{-6x^2+5}\left[(-12x)(1 - 12x^2) - 24x\right] =$$

$$= e^{-6x^2+5}\left[-12x + 144x^3 - 24x\right] =$$

$$e^{-6x^2+5}\left[-36x + 144x^3\right]$$

$$36e^{-6x^2+5}\left[-x + 4x^3\right] = 36x\,e^{-6x^2+5}\left[-1 + 4x^2\right]$$

$$36 \cdot 4 = 24,\ 120,\ 144 \qquad 12 \cdot 12 = 24,\ 120,\ 144$$

quando $f''(x) > 0$?

F_1 $\quad 36xe^{-6x^2+5} > 0$ $\qquad\qquad$ F_2 $\quad -1 + 4x^2 \geq 0$

$36x > 0$

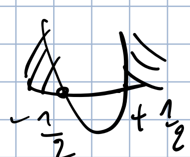
$x > 0$

$4x^2 \geq 1$

$4x^2 = 1$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$



	$-\frac{1}{2}$		0		$\frac{1}{2}$	
F_1	$-$		$-$	$+$		$+$
F_2	$+$		$-$	$-$		$+$
	$\cap$		$\cup$	$\cap$		$\cup$

2. Calcolare l'area A della parte limitata di piano compresa tra i grafici delle funzioni $f(x) = x^2 - 2x$ e $g(x) = -x$.

~~a~~ $A = \frac{1}{6}$ b $A = \frac{1}{3}$ c $A = \frac{9}{2}$ d $A = \frac{1}{2}$ e $A = 6$

quando si incontrano?

$x^2 - 2x = -x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0 \quad x = 1$

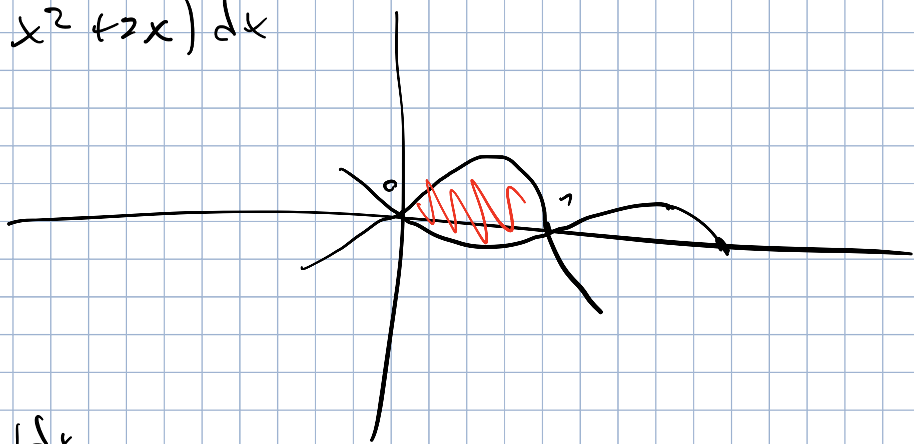
$x = \frac{1}{2}$

$f(\frac{1}{2}) = \frac{1}{2}^2 - 1 = \frac{1}{4} - 1 = -\frac{3}{4}$

$g(\frac{1}{2}) = -\frac{1}{2}$

quindi $\int_0^1 (g(x) - f(x))\,dx$

$$\int_0^1 (-x - x^2 + 2x)\,dx$$



$$\int_0^1 (-x^2 + x)\,dx$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2}\right|_0^1 = -\frac{1}{3} + \frac{1}{2} = \frac{-2+3}{6} = +\frac{1}{6}$$

3. Determinare tutti i valori di α per i quali sia convergente l'integrale improprio

$$\int_1^{+\infty} \frac{\sqrt{1+x^3}}{(4+3x^2)^{2/\alpha}}.$$

a $0 < \alpha < \frac{2}{5}$ b $\alpha < \frac{8}{5}$ c $0 < \alpha$ d $0 < \alpha < \frac{4}{5}$ ~~e~~ $0 < \alpha < \frac{8}{5}$

$$\int_1^{+\infty} \frac{\sqrt{1+x^3}}{(4+3x^2)^{2/\alpha}}\,dx$$

$x \to 1$

$$\lim_{x\to 1} \frac{\sqrt{1+x^3}}{(4+3x^2)^{2/\alpha}} = \frac{\sqrt{2}}{(7)^{2/\alpha}} \quad \text{top..}$$

$x \to +\infty$

$$\lim_{x\to+\infty} \frac{\sqrt{1+x^3}}{\sqrt[\alpha]{(4+3x^2)^2}} \approx \frac{x^{3/2}}{(3x^2)^{2/\alpha}} = \frac{x^{3/2}}{\underbrace{3^{2/\alpha}}_{\text{lui non cambia convergenza}} \cdot x^{4/\alpha}}$$

$$\approx \int_1^{+\infty} \frac{1}{x^{\frac{4}{\alpha} - \frac{3}{2}}}\,dx \quad \to$$

conv. quando:

$$\frac{4}{\alpha} - \frac{3}{2} > 1$$

$$\frac{4}{\alpha} > \frac{5}{2} \qquad 4 > \frac{5\alpha}{2} \qquad \alpha < \frac{8}{5}$$

$\alpha > 0$!! $\to$ per confronto

E

$0 < \alpha < \frac{8}{5}$

4. Calcolare il limite

$$L = \lim_{x\to 0} \frac{\ln(1-x)^3}{2\sqrt[3]{x+8} - 4 + x}.$$

a $L = -\infty$ b $L = -\frac{30}{7}$ c $L = -\frac{18}{7}$ d $L = 0$ e $L = -\frac{3}{5}$

$$\lim_{x\to 0} \frac{3\ln(1-x)}{2\sqrt[3]{x+8} - 4 + x} = \lim_{x\to 0} \frac{3\ln(1-x)}{2\sqrt[3]{8\left(1+\frac{x}{8}\right)} - 4 + x}$$

$$= \lim_{x\to 0} \frac{3\ln(1-x)}{4\left(1+\frac{x}{8}\right)^{1/3} - 4 + x}$$

svil. num

$$3\ln(1-x) \quad x\to 0 \quad \approx 3\left[-x + o(x^3)\right]$$

svil. den.

$$4\left(1+\frac{x}{8}\right)^{1/3} - 4 + x \quad x\to 0 \quad \approx -4 + x + 4\left[1 + \left(\frac{1}{3}\cdot\frac{x}{8}\right) + o(x)\right]$$

$$\approx -4 + x + 4 + \frac{4x}{24} + o(x) = x + \frac{x}{6} + o(x) = \frac{7x}{6} + o(x)$$

$$\text{tutto} \approx \frac{-3x + o(x)}{\frac{7}{6}x + o(x)} \approx \frac{\cancel{x}(-3 + o(1))}{\cancel{x}\left(\frac{7}{6} + o(1)\right)} = -\frac{18}{7}$$

5. Sia f una funzione derivabile otto volte in $\mathbb{R}$ tale che $f(x) = 2x^5 + 3x^7 + o(x^7)$ per $x \to 0$. Allora

a $f^{(8)}(0) = 0$ b $f^{(7)}(0) = 3 \cdot 7!$ c $f^{(5)}(0) = 2$ d $f^{(5)}(0) = \frac{2}{5!}$ e 0 è un punto di minimo

$f^{(8)}(0)$ non la so, $f^{(5)}(0)$ è $2 \cdot 5!$ quindi no "c" ne "d"

$f^{(7)}(0) = 3 \cdot 7!$

6. Calcolare il seguente integrale (suggerimento: eseguire un integrazione per parti)

$$I = \int_0^1 \ln(9 + x^2) - 3 \; dx.$$

[a] $I = \ln 10 - 5 + 6 \arctan 3$ [b] $I = \ln 10 - 4 + 3 \arctan(1/3)$

[c] $I = \ln 10 - 5 + 6 \arctan(1/3)$ [d] $I = \ln 10 - 4 + 3 \arctan 3$

[e] $I = \ln 10 - 3$

$$I = \int_0^1 \ln(9+x^2) - 3 \; dx = \int \underset{g(x)}{1} \underset{f(x)}{\ln(9+x^2)}\,dx - \int 3\,dx =$$

$$\int 1 -$$

$$= x\ln(9+x^2) - \int \frac{2x^2}{9+x^2}\,dx - 3x$$

$$x^2 + 0 + 0 \;\big|\; x^2 + 9$$
$$-x^2 - 9 \quad 1$$
$$-9$$

$$= x\ln(9+x^2) - 2\int \frac{x^2}{9+x^2}\,dx - 3x =$$

$$= x\ln(9+x^2) - 2\left[\int 1\,dx \left[- 9\int \frac{1}{x^2+9}\right]\right] - 3x$$

$$-\frac{9}{9}\left(\frac{1}{\left(\frac{x}{3}\right)^2 + 1}\right) \to -3\left(\frac{\frac{1}{3}}{\left(\frac{x}{3}\right)^2+1}\right)$$
$$= -3\,\mathrm{arctg}\left(\frac{x}{3}\right)$$

$$x\ln(9+x^2) - 5x + 6\,\mathrm{arctg}\,\frac{x}{3}\Big|_0^1$$

$$\ln|10| - 5 + 6\,\mathrm{arctg}\,\frac{1}{3} - 3$$

7. Qual è l'ordine α di infinitesimo in 0 della seguente funzione

$$f(x) = \frac{x^2(e^x - 1)}{\ln(x + 1)} \; ?$$

[a] $\alpha = 2$ [b] $\alpha = 3$ [c] $\alpha = \frac{1}{2}$ [d] $\alpha = 4$ [e] $\alpha = \frac{3}{2}$

$$f(x) = \frac{x^2(e^x - 1)}{\ln(x+1)} \approx \frac{x^2 \cdot \cancel{x}}{\cancel{x}} \to \alpha = 2$$

$x \to 0 \qquad e^x - 1 \approx x$

$\ln(x+1) \approx x$

8. Qual è, tra quelli indicati, lo sviluppo di Taylor di ordine 3 centrato in 0 della funzione

$$f(x) = \sin(x + 2x^2) - 2xe^x \ ?$$

$\boxed{\text{a}}$ $-x + 4x^3 + o(x^3)$

$\boxed{\text{b}}$ $-x - 4x^2 - \frac{x^3}{6} + o(x^3)$

$\boxed{\text{c}}$ $-x - x^2 + o(x^3)$

$\boxed{\text{d}}$ $-x - \frac{7}{6}x^3 + o(x^3)$

$\boxed{\text{e}}$ $-x + x^3 + o(x^3)$

$$f(x) = \operatorname{sen}(x + 2x^2) - 2xe^x$$

$$x + 2x^2 - \frac{x^3}{6} - 2x\left(1 + x + \frac{x^2}{2}\right) = x + \cancel{2x^2} - \frac{x^3}{6} - 2x - \cancel{2x^2} - x^3$$

$$= -x - \frac{7}{6}x^3 + o(x^3)$$